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Indian Standard

STATISTICAL TOLERANCE INTERVAL — METHODS FOR DETERMINATION

UDC 519·233·2

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Statistical Methods for Quality and Reliability Sectional Committee, BSD 1

FOREWORD

This Indian Standard was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Basic Standards, Systems and Services Division Council.

For a quick evaluation of product quality, estimation of the average quality of a lot or process and the range within which most of the values of a variable quality characteristic under investigation will lie, is of great importance.

The methods for determination of the sample size which will enable the estimation of the average quality of a lot or process with a specified degree of accuracy are given in IS 5002 : 1969 'Methods for determination of sample size to estimate the average quality of a lot or process'.

The present standard provides methods for the determination of Statistical Tolerance Interval for a given sample size so that it will be helpful in the evaluation of product quality mentioned above.

If the form of distribution of the quality characteristic, as also the values of the parameters involved, are known, the range within which a given percentage of the observations in a lot or process will lie can be accurately determined. For example if average (μ) and standard deviation (σ) of a normal process are known, it is certain that the range $\mu \pm 3\sigma$ will contain 99·73 percent values of the process.

If, however, one or more of the parameters are to be estimated from a sample, the range set to contain a required percentage of the observations becomes a random variable and varies from sample to sample. If different samples are taken from the same process, some of the ranges (even though constructed using the same formula) may not contain the stipulated percentage of observations.

It is, therefore, desirable to construct an interval (L_1, L_2) with two limits L_1 and L_2, called tolerance interval in such a manner that at least P percentage of the products in a lot or process will lie within this interval, with a specified degree of confidence, say β, no matter what particular sample appears during the time of investigation. In some cases it may be necessary to construct only one-sided intervals, namely, upper tolerance interval ($-\infty$, L_2) or lower tolerance interval (L_1, ∞). To be meaningful both β and P should be high.

In reporting the results of a test or analysis made in accordance with this standard, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS 2 : 1960 'Rules for rounding off numerical values (*revised*)'.

Indian Standard

STATISTICAL TOLERANCE INTERVAL — METHODS FOR DETERMINATION

1 SCOPE

1.1 This standard lays down methods for determination of a statistical tolerance interval for a variable quality characteristic of a product in a lot or process, which is supposed to cover a specified percentage of observations in the population with a stated degree of confidence. These methods are worked out under the assumptions that the sample observations are drawn independently at random and that the process producing the lot is in a state of statistical control.

1.2 This standard provides methods for determining one-sided or two-sided tolerance interval, as the case may be, for a quality characteristic distributed normally. In cases, for which a quality characteristic is not distributed normally, the sample size required to have the same length of tolerance interval containing same percentage of observations and same degree of confidence is enormously large as compared to the normal case. Methods for these cases are also given in this standard, and shall be used only when tests for normality as given in IS 6200 (Part 3) : 1984, fail and if the original variable may not be easily transformed to normal form.

2 REFERENCES

The following Indian Standards are necessary adjuncts to this standard:

IS No.	*Title*
6200 (Part 3) : 1984	Statistical tests of significance: Part 3 Tests for normality
7920 : 1985	Statistical vocabulary (*first revision*)
9300 (Part 2) : 1989	Statistical models for industrial applications: Part 2 Continuous models (*first revision*)

3 TERMINOLOGY

3.1 For the purpose of this standard, the following definitions, in addition to those given in IS 7920 : 1985, shall apply.

3.2 Statistical Tolerance Interval

If two statistics L_1 and L_2 ($L_1 < L_2$) based on sample observations are calculated so that the variable interval (L_1, L_2) includes at least a given percentage P of the observations in a lot or process from which the sample is drawn, with a specified probability β, then the interval is called a statistical tolerance interval of strength (β, P). If for a particular sample the numerical values of L_1 and L_2 are l_1 and l_2 respectively then l_1 and l_2 are called the corresponding lower and upper tolerance limits.

Mathematically, it means that:

Probability [Percentage of observations between l_1 and $l_2 > P$] $= \beta$

Thus a statistical tolerance interval of strength (0·99, 95) will mean that if the tolerance interval is computed for each of a large number of samples drawn from the lot or the process then in 99 percent of the cases the interval thus calculated will include at least 95 percent of the observations in the lot or the process.

If only one limit is formed so that at least P (%) of the observations in the lot or process will lie above (below) the limit with a probability β, then the limit will be called one sided lower (upper) tolerance limit giving rise to one sided tolerance interval.

3.3 Tolerance Factor

The tolerance limits are obtained by adding to and subtracting from the sample mean ($\bar{x}$) a multiple of standard deviation (σ or s). This multiplier is called the tolerance factor and is denoted by λ. Its value depends on the sample size n, confidence coefficient β and the chosen level P. It will also depend on whether standard deviation is known or estimated by sample standard deviation (s). In order to distinguish between the two cases, we shall use the symbol λ_1 and λ_2 respectively for the two cases.

Though the tolerance interval varies from sample to sample, it is to be noted that the tolerance factor remains the same as long as sample size remains same.

4 DETERMINATION OF STATISTICAL TOLERANCE INTERVAL WHEN THE DISTRIBUTION OF THE QUALITY CHARACTERISTIC IS NORMAL

4.1 One-Sided Tolerance Interval

4.1.1 *Lot or Process Variability (σ) is Known*

The upper (lower) tolerance limit that will have at least P (%) of the observations below

(above) it with a probability β (confidence coefficient) is obtained by adding (subtracting) λ_1 times the given σ to (from) the sample mean $\bar{x}$. The values of λ_1 are given in Table 1 for two levels of β as 0·95 and 0·99 and three levels of P as 90%, 95% and 99% for some selected values of n. For cases not covered in Table 1, (n, β, P) can be calculated from the formula given below:

$$\lambda_1 = K_{1-p} + (K_{1-\beta}/\sqrt{n})$$

Where $p = P/100$ and K_{1-p} and $K_{1-\beta}$ are the normal variate values such that the upper tail probabilities are ($1-p$) and ($1-\beta$) respectively. The normal variate values are given in Table 7.

For example, if it is required to find λ_1 (30, 0·99, 95), then

$$p = 0{\cdot}95 \text{ and } \beta = 0{\cdot}99,$$

$$K_{1-p} = 1{\cdot}645 \text{ and } K_{1-\beta} = 2{\cdot}326 \text{ (\textit{see} Table 7).}$$

Then, $\lambda_1 = 1{\cdot}645 + 2{\cdot}326/\sqrt{30} = 2{\cdot}070$

4.1.1.1 *Example*

The mass in kg of 20 steel tubes of 100 mm nominal bore taken at random from a controlled manufacturing process is given below :

10·35 9·65 10·27 10·85 9·72 9·75 10·18 10·26 9·36 10·15
9·60 10·25 9·86 9·45 9·82 10·15 10·75 9·80 10·10 10·08

The standard deviation of the process is 0·4 kg. What will be the 95 percent upper tolerance limit for mass of a steel tube such that 99 percent of the tubes have mass below it ?

It is required to find upper tolerance limit of strength (0·95, 99) based on $n=20$. Referring to Table 1, λ_1 (20, 0·95, 99) is found to be 2·694. The sample mean $\bar{x}$ is 10·02. Hence the upper tolerance limit is:

$$\begin{aligned}\bar{x} + \lambda_1\sigma &= 10{\cdot}02 + 2{\cdot}694 \times 0{\cdot}4 \\ &= 11{\cdot}10 \text{ kg}\end{aligned}$$

Suppose it is required to predict with 95 percent confidence the lower tolerance limit such that 99 percent of the tubes manufactured by the process are heavier than this value. In this case the lower tolerance limit is :

$$\begin{aligned}\bar{x} - \lambda_1\sigma &= 10{\cdot}02 - 2{\cdot}694 \times 0{\cdot}4 \\ &= 8{\cdot}94 \text{ kg}\end{aligned}$$

4.1.2 *Lot or Process Variability* (σ) *is Unknown*

In this case, the one-sided upper tolerance limit is of the form $\bar{x} + \lambda_2 s$ and one-sided lower tolerance limit is of the form $\bar{x} - \lambda_2 s$.

λ_2 is obtained from Table 2 for $\beta = 0{\cdot}95$ and 0·99 and $P=90$ percent, 95 percent and 99 percent and $n = 7(1)\ 20(2)\ 30(5)\ 50(10)\ 100(50)\ 300(100)\ 500$, 1 000 and ∞. For other choices of sample sizes not covered here, one has to interpolate.

4.1.2.1 *Example*

In order to study the efficiency of filling operation in a pharmaceutical plant, twenty 2 cc ampoules were taken at random after filling and the volumes of content were measured in cc. The data are recorded below :

1·96 2·15 1·80 2·11 2·12 2·04 1·89 1·86 2·11 1·88
2·26 2·29 2·03 2·23 2·25 1·80 1·83 2·10 2·05 2·24

The process standard deviation is not known. The management wants to know with 99 percent confidence the volume of medicine that will be exceeded by 99 percent of the ampoules.

It is required to find the lower tolerance limit of strength (0·99, 99) for $n=20$. Referring to Table 2, λ_2 (20, 0·99, 99) is found to be 3·830.

The sample mean, $\bar{x} = 2{\cdot}05$ cc

The sample standard deviation, $s = 0{\cdot}163\ 4$ cc

Hence the lower confidence limit is:

$$\begin{aligned}\bar{x} - \lambda_2 s &= 2{\cdot}05 - 3{\cdot}830 \times 0{\cdot}163\ 4 \\ &= 2{\cdot}05 - 0{\cdot}626 \\ &= 1{\cdot}42 \text{ cc}\end{aligned}$$

In the same example if the management's interest is to know the extent of overfilling one may like to find with 95 percent confidence the upper tolerance limit which will not be exceeded by more than 1 percent of the ampoules.

Here it is required to find the upper tolerance limit of strength (0·95, 99) for $n=20$.

From Table 2, λ_2 (20, 0·95, 99) is found to be 3·295. Sample standard deviation, s is as before.

Hence the required upper tolerance limit is

$$\begin{aligned}\bar{x} + \lambda_2 s &= 2{\cdot}05 + 3{\cdot}295 \times 0{\cdot}163\ 4 \\ &= 2{\cdot}05 + 0{\cdot}538 = 2{\cdot}59 \text{ cc}\end{aligned}$$

4.2 Two-sided Tolerance Interval

4.2.1 *Lot or Process Variability* (σ) *is Known*

The two-sided tolerance interval of strength (β, P) is $\bar{x} \pm \lambda_1\sigma$, where $\bar{x}$ is the sample mean and λ_1 is the tolerance factor, the value of which has been given in Table 3 for $\beta = 0{\cdot}95$ and 0·99 and $P=90$ percent, 95 percent and 99 percent and $n = 5(1)\ 20(2)\ 30(5)\ 50(10)\ 100(50)\ 300(100)\ 500$, 1 000 and ∞. For other choices of sample sizes not covered here, one has to interpolate.

4.2.1.1 *Example*

In the example given in **4.1.1.1**, the 95 percent two-sided tolerance interval for the distribution of mass of steel tubes such that at least 99 percent of the tubes will have mass within this interval is:

$$\begin{aligned}&\bar{x} \pm \lambda_1 (20, 0{\cdot}95, 99).\ s \\ &= (10{\cdot}02 \pm 2{\cdot}789 \times 0{\cdot}4) \text{ kg} \\ &= (10{\cdot}02 \pm 1{\cdot}116) \text{ kg} \\ &= (8{\cdot}90,\ 11{\cdot}14) \text{ kg}\end{aligned}$$

Thus one can assert with 95 percent confidence that 99 percent of the tubes will have their masses between 8·90 kg and 11·14 kg.

4.2.2 *Lot or Process Variability* (σ) *is Unknown*

The two-sided tolerance interval of strength

(β, P) is $\bar{x} \pm \lambda_2 s$ where $\bar{x}$ and s are respectively the sample mean and standard deviation and λ_2 is the tolerance factor of the required strength. The value of λ_2 for several choices of n, β and P is given in Table 4. For other choices of these parameters, the value of λ_2 (n, β, P) can be obtained from the formula:

$$\lambda_2 = K_{(1-p)/2} \, . \, \sqrt{n/D}$$

where $D = (n-1) - \sqrt{2 (n-1)}\, K_1 + \frac{2}{3} (K^2{}_{1-\beta} - 1) + \frac{(7K_{1-\beta} - K^3{}_{1-\beta})}{9\sqrt{2 (n-1)}}$

and $p = P/100$

and $K_{(1-p)/2}$ and $K_{1-\beta}$ are normal variate values (*see* Table 7) such that the upper tail probabilities are ($1-p$)/2 and (1 − β) respectively.

For example, in order to find for a sample of size $n=26$ from a process whose standard deviation is unknown, the tolerance factor which will give a two-sided interval including 99 percent of the cases one has to compute λ_2 (26, 0·95, 99).

Hence $n=26$, $p=0{\cdot}99$, $\beta=0{\cdot}95$, so that

$K_{(1-p)/2} = 2{\cdot}576$

$K_{1-\beta} = 1{\cdot}645$

$D = 14{\cdot}617$

Hence λ_2 (26, 0·95, 99) $= 2{\cdot}576 \times \sqrt{26/14{\cdot}617}$

$= 2{\cdot}576 \times 1{\cdot}334$

$= 3{\cdot}436$

4.2.2.1 *Example*

A sample of 30 observations was collected to record the weight of a sliver piece of 6 metre length at the drawing stage of a Textile mill. The weights in grains are given below :

315 316 320 318 319 320 321 322 310 308
312 315 320 316 325 309 318 316 322 318
314 316 321 312 320 315 310 309 312 316

In order to control the consumption of raw cotton the management is interested to know for certain, the interval within which 99 percent of the observations will lie if the current level of consumption is maintained.

Since the process mean μ and standard deviation σ are unknown, the interval cannot be predicted with certainty. However, if the management is satisfied to a prediction of the interval with a confidence of 0·95 instead of 1, the interval can be worked out.

In this case it is required to obtain a two-sided tolerance interval with $\beta=0{\cdot}95$ and $P=99$

The value of λ_2(30, 0·95, 99) is read as 3·352 from Table 4.

$\bar{x} = 316{\cdot}17$ grains

$s = 4{\cdot}46$ grains

Hence the desired tolerance interval is :

$\bar{x} \pm \lambda_2 s = (316{\cdot}17 \pm 3{\cdot}352 \times 4{\cdot}46)$ grains

$= (301, 331)$ grains

5 DETERMINATION OF STATISTICAL TOLERANCE INTERVAL WHEN THE DISTRIBUTION OF QUALITY CHARACTERISTIC IS NOT NORMAL

5.1 Introduction

In a situation when the distribution of the quality measure of an item is markedly non-normal, as evident from a plot on the normal probability paper or application of statistical tests as given in IS 6200 (Part 3) : 1984, the procedures for obtaining tolerance interval as outlined in **4** will not be valid. In this case, the tolerance interval of strength (β, P) may be given by a single order statistic $x_{(r)}$ in case of one-sided interval or a pair of symmetrically placed order statistics $x_{(r)}$, $x_{(s)}$ with $s=n_{-r+1}$ in case of two-sided interval where $x_{(k)}$ is the k-th smallest observation in a random sample of size n taken from the population.

To keep the sample size n at a manageable level, r is normally taken as 1, so that one or both of the extreme values $X_{(1)}$ (the smallest value) and $X_{(n)}$ (the largest value) in the samples are used to give the required tolerance interval.

5.2 One-sided Tolerance Interval

In this case, one-sided tolerance interval of strength (β, P) is given by [$X_{(1)}$. ∞] or [−∞, $X_{(n)}$]. If P (%) of the population values are required to exceed (fall short of) the smallest observation $X_{(1)}$ [the largest observation $X_{(n)}$] in a random sample of size n with the probability at least β, then n is given as the smallest integer satisfying :

$$p^n \leqslant 1 - \beta$$

where $p = P/100$

Thus given P and β, one can determine, the minimum sample size n which will ensure that at least P (%) of the population values will be larger (smaller) than the smallest (largest) observation in a sample of that size.

Values of n satisfying the above relation for p and β equal to 0·50, 0·75, 0·90, 0·95, 0·975, 0·99 and 0·999 are given in Table 5.

5.3 Two-sided Tolerance Interval

Here two-sided tolerance interval of strength (β, P) is given by [$X_{(1)}$, $X_{(n)}$]. If at least P (%) of the population values are required to be covered by the smallest observation $X_{(1)}$ and the largest observation $X_{(n)}$ in a sample of size n, with probability at least β, the minimum sample size required to ensure this is the smallest integer n satisfying:

$$np^{n-1} - (n - 1) p^n \leqslant 1 - \beta$$

where $p = P/100$

From the above relation, one may also determine p for given n and β, or β for given n and p. Values of n satisfying this relation for p and β equal to 0·50, 0·75, 0·90, 0·95, 0·975, 0·99, 0·999 are given in Table 5.

5.4 Example

The following data relate to life (in hours) for a random sample of 30 electric tubes of a certain brand:

2 886	2 647	2 404	785	1 277	1 345
877	1 597	1 314	634	1 083	1 260
1 373	808	1 578	934	1 363	2 759
1 184	600	1 684	1 963	834	912
1 453	855	699	942	997	1 731

a) What is the minimum percentage of tubes in the entire lot whose life will exceed 600 hours (fall short of 2 886 hours) with probability at least 0·95 ?

Here $n = 30$, $X_{(1)} = 600$, $X_{(30)} = 2\ 886$ and $\beta = 0·95$

The expression given in **5.2**, with equality sign, gives for $n = 30$ and $\beta = 0·95$, a value of P slightly higher than 90. Hence, at least 90 percent of the tubes will have life each exceeding 600 hours (less than 2 886 hours).

b) What sample size must be taken to make it possible to assert at a confidence level 0·95 that at least 99 percent of the tubes in the lot are from a controlled process, will have life not exceeding the largest observation (at least equal to the smallest observation) in the sample ?

Here $\beta = 0·95$, $p = 0·99$

Table 5 gives $n = 299$

c) What is the minimum percentage of tubes in the entire lot whose life will be between 600 h and 2 886 h with probability 0·999.

Here $\beta - 0·999$, $n = 30$

It can be seen from Table 6 that the required minimum percentage is slightly less than 75 percent.

d) What sample size must be taken to make it possible to assert at a confidence level 0·95 that at least 99 percent of the tubes will have their lives between the smallest and the largest observation in the sample ?

For $\beta = 0·95$ and $p = 0·99$, the required sample size from Table 6 is obtained as 473.

6 REMARKS

6.1 The tolerance interval given by the tolerance limits $\bar{x} \pm \lambda_1\sigma$ or $\bar{x} \pm \lambda_2 s$ is of length $2\lambda_1\sigma$ or $2\lambda_2 s$. For a given choice of β and P the length of the interval depends on and decreases with sample size. The tolerance interval is rendered trivial if the tolerance factor (λ_2) is very large. Since crucial decisions are taken on the basis of statistical tolerance interval, it is recommended that the 'sample size n should be reasonably large if the cost of sampling and inspection is not prohibitive. Usually $n > 20$ should be preferred.

6.2 The determination of and conclusions drawn from tolerance interval are valid only when the process is under a state of statistical control. Once the sample is drawn, sample observations should not be removed or replaced unless there is an evidence that it was the outcome of an assignable cause not to occur again.

6.3 Tolerance limits of the form $\bar{x}\pm\lambda_2 s$ are not to be confused with confidence limits for the average quality of a lot or mean of a process which are also of the form $\bar{x}\pm Ks$. Confidence limits for the mean are computed in such a way that it can be asserted with β degree of confidence that the unknown average lot quality or process mean will lie between these two limits.

In contrast, tolerance limits are constructed in such a way that it can be asserted with β degree of confidence that the interval will include at least P (%) of the observations in the lot or process. Other things remaining same, the value of λ_2 and K both decrease as n increases. As n becomes larger and larger, K diminishes towards zero where as λ_2 diminishes towards the upper ($100 - P$)/2 percent point of the normal deviate so that the interval includes P% of the population observations. Thus, for any β, if P is 95,

$$\lambda_2 \longrightarrow 1·96 \text{ as } n \longrightarrow \infty$$

$$\text{Where as } K \longrightarrow 0 \quad \text{as } n \longrightarrow \infty$$

Table 1 Values of Tolerance Factor λ_1 for One-sided Statistical Tolerance Interval (σ Known)

(*Clauses* 4.1.1 *and* 4.1.1.1)

β→	0·95			0·99		
P(%)→	90	95	99	90	95	99
n↓						
5	2·017	2.380	3·062	2·322	2·685	3·367
6	1·953	2·316	2·998	2·231	2.595	3·276
7	1·903	2·267	2·948	2·161	2·524	3 206
8	1·863	2·226	2·908	2·104	2·467	3·149
9	1·830	2·193	2·875	2·057	2·420	3·102
10	1 ·802	2·165	2·846	2·017	2·381	3 062
11	1·777	2·141	2·822	1·983	2·346	3·028
12	1·756	2·120	2·801	1·953	2·316	2·998
13	1·738	2·101	2·783	1·927	2·290	2·972
14	1·721	2·084	2·766	1·903	2·267	2·948
15	1·706	2·070	2·751	1·882	2·246	2 927
16	1·693	2·056	2·738	1·863	2·226	2·908
17	1·680	2·044	2·725	1·846	2·209	2·890
18	1·669	2·030	2·714	1·830	2·193	2·875
19	1·659	2·022	2·704	1·815	2·179	2·860
20	1·649	2·013	2·694	1·802	2·165	2·846
22	1·632	1·996	2·677	1·778	2·141	2·822
24	1·617	1·981	2·662	1·756	2·120	2·801
26	1·604	1·967	2·649	1·738	2·101	2·782
28	1·592	1·956	2·637	1·721	2·084	2·766
30	1·582	1·945	2·627	1·706	2·070	2·751
35	1·560	1·923	2·604	1·675	2·038	2·720
40	1·542	1·905	2·586	1·649	2·013	2·694
45	1·527	1·890	2·572	1·628	1·992	2·673
50	1·514	1·877	2·559	1·611	1·974	2·655
60	1·494	1·857	2·539	1·582	1·945	2·627
70	1·478	1·841	2·523	1·560	1·923	2·604
80	1·465	1·829	2·510	1·542	1·905	2·586
90	1·455	1·818	2·500	1·527	1·890	2·571
100	1·446	1·809	2·491	1·514	1·877	2·559
150	1·416	1·779	2·461	1·471	1·835	2·516
200	1·398	1·761	2·443	1·446	1·809	2·491
250	1·386	1·749	2·430	1·429	1·792	2·473
300	1·377	1·740	2·421	1·416	1·779	2·461
400	1·364	1·727	2·409	1·398	1·761	2·443
500	1·355	1·718	2·400	1·386	1·749	2·430
1000	1·333	1·697	2·378	1·355	1·718	2·400
∞	1·282	1·645	2·326	1·282	1·645	2·326

Table 2 Values of Tolerance Factor λ_2 for One-sided Statistical Tolerance Interval (σ Unknown)

(*Clauses* 4.1.2 *and* 4.1.2.1)

β→	0·95			0·99		
P(%)→	90	95	99	90	95	99
n↓						
7	2·756	3·400	4·643	3·886	4·751	6·427
8	2·582	3·188	4·354	3·483	4·283	5·831
9	2·453	3·032	4·144	3·232	3·973	5·388
10	2·355	2·911	3·981	3·048	3·738	5·075
11	2·275	2·814	3·852	2·896	3·554	4·827
12	2·209	2·735	3·746	2·774	3·407	4·630
13	2·155	2·670	3·658	2·674	3·286	4·469
14	2·108	2·714	2·584	2·591	3·186	4·335
15	2·068	2·565	3·519	2·520	3·100	4·221
16	2.033	2·523	3·464	2·459	3·027	4·122
17	2·002	2·486	3·414	2·405	2·962	4·037
18	1·974	2·453	3·370	2·356	2·904	3·960
19	1·948	2·423	3·330	2·313	2·853	3·891
20	1·926	2·396	3·295	2·274	2·806	3·830
22	1·886	2·349	3·233	2·208	2·727	3·725
24	1·853	2·309	3·181	2·152	2·661	3·638
26	1·824	2·275	3·136	2·105	2·605	3.64
28	1·799	2·245	3·097	2·065	2·557	3·501
30	1·777	2·220	3·064	2·029	2·515	3·545
						4
35	1·732	2·167	2·994	1·957	2·430	3·334
40	1·697	2·125	2·941	1·901	2·364	3·248
45	1·669	2·092	2·898	1·857	2·311	3·180
50	1·645	2·065	2·862	1·820	2·268	3·124
60	1·609	2·022	2·807	1·764	2·202	3·038
70	1·581	1·990	2·765	1·721	2·152	2·973
80	1·559	1·964	2·732	1·688	2·113	2·923
90	1·542	1·944	2·706	1·661	2·082	2·883
100	1·527	1·926	2·684	1·639	2·056	2·849
150	1·478	1·870	2·611	1·566	1·971	2·740
200	1·450	1·837	2·570	1·524	1·923	2·678
250	1·431	1·815	2·542	1·496	1·891	2·637
300	1·417	1·800	2·522	1·476	1·867	2·608
400	1·398	1·778	2·494	1·448	1·836	2·567
500	1·385	1·763	2·475	1·430	1·814	2·540
1 000	1·354	1·727	2·430	1·385	1·762	2·474
∞	1·281	1·645	2·326	1·281	1·645	2·326

Table 3 Values of Tolerance Factor λ_1 for Two-sided Statistical Tolerance Interval (σ Known)

(*Clause* 4.2.1)

β→	0·95			0·99		
P(%)→	90	95	99	90	95	99
n ↓						
5	2·165	2·525	3·205	2·435	2·798	3·479
6	2·093	2·451	3·130	2·336	2·699	3·379
7	2·039	2·395	3·073	2·259	2·621	3·301
8	1·996	2·350	3·024	2·198	2·559	3·239
9	1·961	2·313	2·986	2·148	2·508	3·187
10	1-933	2·283	2·954	2·107	2·465	3·144
11	1·909	2·258	2·926	2·071	2·429	3·108
12	1·889	2·236	2·903	2·041	2·397	3·075
13	1·872	2·218	2·882	2·015	2·370	3·046
14	1·857	2·201	2·864	1·992	2·346	3·019
15	1·844	2·187	2·848	1·971	2·324	2·997
16	1·832	2·174	2·834	1·953	2·305	2·977
17	1·822	2·163	2·821	1·937	2·288	2·958
18	1·812	2·152	2·809	1·922	2·272	2·941
19	1·804	2·143	2·799	1·909	2·258	2·926
20	1·797	2·135	2·789	1·897	2·245	2·912
22	1·785	2·122	2·774	1·878	2·225	2·890
24	1·773	2·109	2·759	1·859	2·204	2·867
26	1·763	2·098	2·745	1·844	2·187	2·848
28	1·755	2·089	2·735	1·831	2·173	2·833
30	1·748	2·080	2·725	1·818	2·159	2·817
35	1·733	2·063	2·706	1·795	2·133	2·787
40	1·723	2·051	2·691	1·777	2·113	2·764
45	1·714	2·041	2·679	1·763	2·097	2·745
50	1·708	2·033	2·669	1·751	2·084	2·730
60	1·697	2·022	2·655	1·734	2·064	2·707
70	1·690	2·013	2·644	1·722	2·050	2·689
80	1·684	2·007	2·636	1·712	2·039	2·676
90	1·680	2·002	2·629	1·705	2·031	2·666
100	1·677	1·998	2·624	1·699	2·024	2·657
150	1·666	1·985	2·609	1·681	2·003	2·631
200	1·661	1·979	2·601	1·672	1·993	2·618
250	1·658	1·975	2·596	1·667	1·986	2·610
300	1·656	1·973	2·593	1·663	1·982	2·604
400	1·653	1·970	2·589	1·659	1·977	2·597
500	1·652	1·968	2·586	1·656	1·973	2·593
1 000	1·649	1·964	2·581	1·651	1·967	2·585
∞	1·645	1·960	2·576	1·645	1·960	2·576

Table 4 Values of Tolerance Factor λ_2 for Two-sided Statistical Tolerance Interval (σ Unknown)

(*Clauses* 4.2.2 *and* 4.2.2.1)

β→	0·95			0·99		
P(%)→	90	95	99	90	95	99
n ↓						
5	4·275	5·079	6·634	6·612	7·855	10·260
6	3·712	4·414	5·775	5·337	6·345	8·301
7	3·369	4·007	5·248	4·613	5·488	7·187
8	3·136	3·732	4·891	4·148	4·936	6·468
9	2·967	3·532	4·631	3·822	4·550	5·966
10	2·839	3·379	4·433	3·582	4·265	5·594
11	2·737	3·259	4·277	3·397	4·045	5·308
12	2·655	3·162	4·150	3·250	3·870	5·079
13	2·587	3·081	4·044	3·130	3·727	4·893
14	2·529	3 012	3·955	3·029	3·608	4·737
15	2·480	2·954	3·878	2·945	3·507	4·605
16	2·437	2·903	3·812	2·872	3·421	4·492
17	2·400	2·858	3·754	2·808	3·345	4·393
18	2·369	2·322	3·709	2·748	3·275	4·304
19	2·339	2·787	3·662	8·700	3·217	4·228
20	2·312	2·755	3·620	2·657	3·166	4·160
22	2·265	2·699	3·547	2·582	3·077	4·043
24	2·227	2·653	3·487	2·520	3·003	3·947
26	2·194	2·614	3:435	2·468	2·941	3·865
28	2·165	2·580	3·391	2·423	2·887	3·795
30	2·141	2·551	3·352	2·384	2·841	3·734
35	2·090	2·491	3·274	2·306	2·748	3·611
40	2·052	2·445	3·213	2·247	2·677	3·518
45	2·022	2·409	3·166	2·200	2·621	3·444
50	1·997	2·379	3·127	2·162	2·576	3·385
60	1·958	2·333	3·066	2·103	2·506	3·293
70	1.929	2·299	3·021	2·060	2·454	3·225
80	1·907	2·272	2·986	2·026	2·414	3·173
90	1·889	2·251	2·958	1·999	2·382	3·130
100	1·874	2·233	2·934	1·977	2·356	3·096
150	1·825	2·175	2·859	1·905	2·270	2·984
200	1·798	2·143	2·816	1·865	2·222	2·921
250	1·780	2·121	2·788	1·839	2·191	2·879
300	1·767	2·106	2·767	1·820	2·169	2·850
400	1·749	2·084	2·739	1·794	2·138	2·809
500	1·737	2·070	2·721	1·777	2·117	2·782
1 000	1·709	2·036	2·675	1·736	2·068	2·718
∞	1·645	1·960	2·576	1·645	1·960	2·576

Table 5 Minimum Sample Size n for One-sided Tolerance Interval ($X_{(1)}$, ∞) or ($-\infty$, $X_{(n)}$) to Ensure Strength (β, P)

(*Clauses* 5.2 *and* 5.4)

P%→ β↓	50·0	75·0	90·0	95·0	97·5	99·0	99·9
0·500	1	3	7	14	28	69	693
0·750	2	5	14	28	55	138	1 386
0·900	4	9	22	45	91	230	2 302
0·950	5	11	29	59	119	299	2 945
0·975	6	13	36	72	146	368	3 688
0·990	7	17	44	90	182	459	4 603
0·999	10	25	66	135	273	688	6 905

Table 6 Minimum Sample Size n for Two-sided Tolerance Interval ($X_{(1)}$, $X_{(n)}$) to Ensure Strength (β, P)

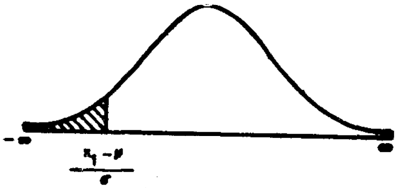
(*Clauses* 5.3 *and* 5.4)

P%→ β↓	50·0	75·0	90·0	95·0	97·5	99·0	99·9
0·500	3	7	17	34	67	168	1 679
0·750	5	10	27	53	107	269	2 692
0·900	7	15	38	77	155	388	3 889
0·950	8	18	46	93	188	473	4 742
0·975	9	20	54	110	221	554	5 569
0·990	11	24	64	130	262	662	6 636
0·999	14	33	89	181	366	920	9 230

Table 7 Areas Under the Normal Curve

(*Clauses* 4.1.1 *and* 4.2.2)

Proportion of total area under the curve left of $\frac{x_1-\mu}{\sigma}$ (x_1 represents any desired value of the variable x).



NORMAL CURVE

The shaded portion is the area which is given in the table.

$\frac{x_1-\mu}{\sigma}$	0·09	0·08	0·07	0·06	0·05	0·04	0·03	0·02	0·01	0·00
−3·5	0·000 17	0·000 17	0·000 18	0·000 19	0·000 19	0·000 20	0·000 21	0·000 22	0·000 22	0·000 23
−3·4	0·000 24	0·000 25	0·000 26	0·000 27	0·000 28	0·000 29	0·000 30	0·000 31	0·000 33	0·000 34
−3·3	0·000 35	0·000 36	0·000 38	0·000 39	0·000 40	0·000 42	0·000 43	0·000 45	0·000 47	0·000 48
−3·2	0·000 50	0·000 52	0·000 54	0·000 56	0·000 58	0·000 60	0·000 62	0·000 64	0·000 66	0·000 69
−3·1	0·000 71	0·000 74	0·000 76	0·000 79	0·000 82	0·000 85	0·000 87	0·000 90	0·000 94	0·000 97
−3·0	0·001 00	0·001 04	0·001 07	0·001 11	0·001 14	0·001 18	0·001 22	0·001 26	0·001 31	0·001 35
−2·9	0·001 4	0·001 4	0·001 5	0·001 5	0·001 6	0·001 6	0·001 7	0·001 7	0·001 8	0·001 9
−2·8	0·001 9	0·002 0	0·002 1	0·002 1	0·002 2	0·002 3	0·002 3	0·002 4	0·002 5	0·002 6
−2·7	0·002 6	0·002 7	0·002 8	0·002 9	0·003 0	0·003 1	0·003 2	0·003 3	0·003 4	0·003 5
−2·6	0·003 6	0·003 7	0·003 8	0·003 9	0·004 0	0·004 1	0·004 3	0·004 4	0·004 5	0·004 7
−2·5	0·004 8	0·004 9	0·005 1	0·005 2	0·005 4	0·005 5	0·005 7	0·005 9	0·006 0	0·006 2
−2·4	0·006 4	0·006 6	0·006 8	0·006 9	0·007 1	0·007 3	0·007 5	0·007 8	0·008 0	0·008 2
−2·3	0·008 4	0·008 7	0·008 9	0·009 1	0·009 4	0·009 6	0·009 9	0·010 2	0·010 4	0·010 7
−2·2	0·011 0	0·011 3	0·011 6	0·011 9	0·012 2	0·012 5	0·012 9	0·013 2	0·013 6	0·013 9
−2·1	0·014 3	0·014 6	0·015 0	0·015 4	0·015 8	0·016 2	0·016 6	0·017 0	0·017 4	0·017 9
−2·0	0·018 3	0·018 8	0·019 2	0·019 7	0·020 2	0·020 7	0·021 2	0·021 7	0·022 2	0·022 8
−1·9	0·023 3	0·023 9	0·024 4	0·025 0	0·025 6	0·026 2	0·026 8	0·027 4	0·028 1	0·028 7
−1·8	0·029 4	0·030 1	0·030 7	0·031 4	0·032 2	0·032 9	0·033 6	0·034 4	0·035 1	0·035 9
−1·7	0·036 7	0·037 5	0·038 4	0·039 2	0·040 1	0·040 9	0·041 8	0·042 7	0·043 6	0·044 6
−1·6	0·045 5	0·046 5	0·047 5	0·048 5	0·049 5	0·050 5	0·051 6	0·052 6	0·053 7	0·054 8
−1·5	0·055 9	0·057 1	0·058 2	0·059 4	0·060 6	0·061 8	0·063 0	0·064 3	0·065 2	0·066 8
−1·4	0·068 1	0·069 4	0·070 8	0·072 1	0·073 5	0·074 9	0·076 4	0·077 8	0·079 3	0·080 8
−1·3	0·082 3	0·083 8	0·085 3	0·086 9	0.088 5	0·090 1	0·091 8	0·093 4	0·095 1	0·096 8
−1·2	0·098 5	0·100 3	0·102 0	0·103 8	0·105 7	0·107 5	0·109 3	0·111 2	0·113 1	0·115 1
−1·1	0·117 0	0·119 0	0·121 0	0·123 0	0·125 1	0·127 1	0·129 2	0·131 4	0·133 5	0·135 7
−1·0	0·137 9	0·140 1	0·142 3	0·144 6	0·146 9	0·149 2	0·151 5	0·153 9	0·156 2	0·158 7
−0·9	0·161 1	0·163 5	0·166 0	0·168 5	0·171 1	0·173 6	0·176 2	0·178 8	0·181 4	0·184 1
−0·8	0·186 7	0·189 4	0·192 2	0·194 9	0·197 7	0·200 5	0·203 3	0·206 1	0·209 0	0·211 9
−0·7	0·214 8	0·217 7	0·220 7	0·223 6	0·226 6	0·229 7	0·232 7	0·235 8	0·238 9	0·242 0
−0·6	0·245 1	0·248 3	0·251 4	0·254 6	0·257 8	0·261 1	0·264 3	0·267 6	0.270 9	0·274 3
−0·5	0·277 6	0·281 0	0·284 3	0·287 7	0·291 2	0·294 6	0·298 1	0·305 1	0·305 0	0·308 5
−0·4	0·312 1	0·315 6	0·319 2	0·322 8	0·326 4	0·330 0	0·333 6	0·337 2	0·340 9	0·344 6
−0·3	0·348 3	0·352 0	0·355 7	0·359 4	0·363 2	0·366 9	0·370 7	0·374 5	0·378 3	0·382 1
−0·2	0·385 9	0·389 7	0·393 6	0·397 4	0·401 3	0·405 2	0.409 0	0·412 9	0·416 8	0·420 7
−0·1	0·424 7	0·428 6	0·432 5	0·436 4	0·440 4	0·444 3	0·448 3	0·452 2	0·456 2	0·460 2
−0·0	0·464 1	0·468 1	0·472 1	0·476 1	0·480 1	0·484 0	0·488 0	0·492 0	0·496 0	0·500 0

Table 7 (*Concluded*)

$\frac{x_1-\mu}{\sigma}$	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
+0·0	0·500 0	0·504 0	0·508 0	0·512 0	0·516 0	0·519 9	0·523 9	0·527 9	0·531 9	0·535 9
+0·1	0·539 8	0·543 8	0·547 8	0·551 7	0·555 7	0·559 6	0·563 6	0·567 5	0·571 4	0·575 3
+0·2	0·579 3	0·583 2	0·587 1	0·591 0	0·594 8	0·598 7	0·602 6	0·606 4	0·610 3	0·614 1
+0·3	0·617 9	0·621 7	0·625 5	0·629 3	0·633 1	0·636 8	0·640 6	0·644 3	0·648 0	0·651 7
+0·4	0·655 4	0·659 1	0·662 8	0·666 4	0·670 0	0·673 6	0·677 2	0·680 8	0·684 4	0·687 9
+0-5	0·691 5	0·695 0	0·698 5	0·701 9	0·705 4	0·708 8	0·712 3	0·715 7	0·719 0	0·722 4
+0·6	0·725 7	0·729 1	0·732 4	0·735 7	0·738 9	0·742 2	0·745 4	0·748 6	0·751 7	0·754 9
+0·7	0·758 0	0·761 1	0·764 2	0·767 3	0·770 4	0·773 4	0·776 4	0·779 4	0·782 3	0·785 2
+0·8	0·788 1	0·791 0	0·793 9	0·796 7	0·799 5	0·802 3	0·805 1	0·807 9	0·810 6	0·813 3
+0·9	0·815 9	0·818 6	0·821 2	0·823 8	0·826 4	0·828 9	0·831 5	0·834 0	0·836 5	0·838 9
+1·0	0·841 3	0·843 8	0·846 1	0·848 5	0·850 8	0·853 1	0·855 4	0·857 7	0·859 9	0·862 1
+1·1	0·864 3	0·866 5	0·868 6	0·870 8	0·872 9	0·874 9	0·877 0	0·879 0	0·881 0	0·883 0
+1·2	0·884 9	0·886 9	0·888 8	0·890 7	0·892 5	0·894 4	0·896 2	0·898 0	0·899 7	0·901 5
+1·3	0·903 2	0·904 9	0 907 6	0·908 2	0·909 9	0·911 5	0·913 1	0·914 7	0·916 2	0·917 7
+1·4	0·919 2	0·920 7	0·922 2	0·923 6	0·925 1	0·926 5	0·927 9	0·929 2	0·930 6	0·931 9
+1·5	0·933 2	0·934 5	0·935 7	0·937 0	0·938 2	0·939 4	0·940 6	0·941 8	0·942 9	0·944 1
+1·6	0·945 2	0·946 5	0·947 4	0·948 4	0·949 5	0·950 5	0·951 5	0·952 5	0·953 5	0·954 5
+1·7	0·955 4	0·956 4	0·957 3	0·958 2	0·959 1	0·959 9	0 960 8	0·961 6	0·962 5	0·963 3
+1·8	0·964 1	0·964 9	0·965 6	0·966 4	0·967 1	0·967 8	0·968 6	0·969 3	0·969 9	0·970 6
+1·9	0·971 3	0·971 9	0·972 6	0·973 2	0·973 8	0·974 4	0·975 0	0·975 6	0·976 1	0·976 7
+2·0	0·977 3	0·977 8	0·978 3	0·978 8	0·979 3	0·979 8	0·980 3	0·980 8	0·981 2	0·981 7
+2·1	0·982 1	0·983 6	0·983 0	0·983 4	0·983 8	0·984 2	0·984 6	0·985 0	0·985 4	0·985 7
+2·2	0·986 1	0·986 4	0·986 8	0·987 1	0·987 5	0·987 8	0·988 1	0·988 4	0·988 7	0·989 0
+2·3	0·989 3	0·989 6	0·989 8	0·990 1	0·990 4	0·990 6	0·990 9	0·991 1	0·991 3	0·991 6
+2.4	0·991 8	0·992 0	0·992 2	0·992 5	0·992 7	0·992 9	0·993 1	0·993 2	0·993 4	0·993 6
+2·5	0·993 8	0·994 0	0·994 1	0·994 3	0·994 5	0·994 6	0·964 8	0·994 9	0·995 1	0·995 2
+2·6	0·995 3	0·995 5	0·995 6	0·995 7	0·995 9	0·996 0	0·996 1	0·996 2	0·996 3	0·996 4
+2·7	0·996 5	0·996 6	0·996 7	0·996 8	0·996 9	0·997 0	0·997 1	0·997 2	0·997 3	0·997 4
+2·8	0·997 4	0·997 5	0·997 6	0·997 7	0·997 7	0·997 8	0·997 9	0·997 9	0·998 0	0·998 1
+2·9	0·998 1	0·998 2	0·998 3	0·998 3	0·998 4	0·998 4	0·998 5	0·998 5	0·998 6	0·998 6
+3·0	0·998 65	0·998 69	0·998 74	0·998 78	0 998 82	0·998 86	0·998 89	0·998 93	0·998 96	0·999 00
+3·1	0·999 03	0·999 06	0·999 10	0·999 13	0·999 15	0·999 18	0·999 21	0 999 24	0·999 26	0·999 29
+3·2	0·999 31	0·999 34	0·999 36	0·999 38	0·999 40	0·999 42	0·999 44	0·999 46	0·999 48	0·999 50
+3·3	0·999 52	0·999 53	0·999 55	0·999 57	0·999 58	0·999 60	0·999 61	0 999 62	0·999 64	0·999 65
+3·4	0 999 66	0·999 67	0·999 69	0·999 70	0·999 71	0·999 72	0·999 73	0 999 74	0·999 75	0·999 76
+3·5	0·999 77	0·999 78	0·999 78	0·999 79	0·999 80	0·999 81	0·999 81	0·999 82	0·999 53	0·999 83

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